

**Quiz 2 – 9/7/2022**

**Instructions.** You have 15 minutes to complete this quiz. You may use your plebe-issue calculator. You may not use any other materials (e.g., notes, homework, website).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1	1	
2	1	
3	1	
4	1	
5	1	
Total		/ 50

For Problems 1 and 2, consider the following setting.

As an analyst at the Markov Company, you have been tasked with better understanding the performance of the company's high-speed computer network. There are only two types of messages sent on the network: 100-byte messages and 10000-byte messages.

Let  $X$  be the travel time of a message on the company's network in seconds, and let  $Y$  be the size of a message in bytes. Based on historical data, you have determined the joint pmf between  $X$  and  $Y$ :

$p_{XY}$		$Y$	
		100	10000
$X$	1	8/30	0
	10	6/30	1/30
	1	4/30	2/30
	10	2/30	3/30
	100	0	4/30

**Problem 1.** What is the probability that a message has a travel time of 1 second?

[See Example 1 in Lesson 3 for a similar example. Be careful with arithmetic!](#)

**Problem 2.** What is the probability that the message is 10000 bytes long, given that its travel time is 1 second?

[See Example 3 in Lesson 3 for a similar example. Be careful when applying the definition of conditional probability.](#)

For Problems 3, 4 and 5, consider the following setting.

Another analyst at the Markov Company has collected some other data on the company's network performance. According to her data,  $3/4$  of the messages sent on the network are 100 bytes long, and  $1/4$  are 10000 bytes long. In addition, she found that  $1/5$  of 100-byte messages have a travel time of 1 second, while  $2/5$  of 10000-byte messages have a travel time of 1 second.

Let  $X$  be the travel time of a message on the company's network in seconds, and let  $Y$  be the size of a message in bytes.

**Problem 3.** What is the probability that a message has a travel time of 1 second?

Review the law of total probability in Lesson 3. See Example 6 in Lesson 3 for a similar example.

**Problem 4.** Are  $X$  and  $Y$  independent? Give a numerical argument for why or why not.

You can use the definition of independence, or you might find it easier to use one of the facts about independence and conditional probabilities at the bottom of page 3 of Lesson 3.

Also, be careful with how you interpret the probabilities given to you in the problem. Does “ $1/5$  of 100-byte messages have a travel time of 1 second” correspond to  $\Pr\{X = 1 \text{ and } Y = 100\}$  or  $\Pr\{X = 1 \mid Y = 100\}$ ?

**Problem 5.** What is the expected size of a message in bytes?

Review the definition of expected value in Lesson 2. Remember that intuitively, the expected value is a weighted average. What is the (weighted) average message size?